“Quine’s Indispensability Argument”

Chapter 1, Part 1 of *Numbers without Science*

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§1.1: Quine’s Argument

Quine nowhere presents a detailed indispensability argument, though he alludes to one in many places.[1] In Part 1 of this chapter, I first present a concise version of the argument, and proceed to discuss Quine’s defenses of each premise. I indicate how the argument relies on various aspects of Quine’s methodology, including his procedure for determining ontic commitment, physicalism, and confirmation holism. Lastly, I defend the argument against modal reinterpretations of mathematics. In Part 2 of the chapter, I examine Hartry Field’s response to Quine.

Quine’s indispensability argument can be stated rather simply:

(QI) QI.1: We should believe the theory which best accounts for our empirical experience.
QI.2: If we believe a theory, we must believe in its ontic commitments.
QI.3: The ontic commitments of any theory are the objects over which that theory first-order quantifies.
QI.4: The theory which best accounts for our empirical experience quantifies over mathematical objects.
QI.C: We should believe that mathematical objects exist.

The conclusion of Quine’s indispensability argument is thus that as far as we know, mathematical objects exist. Our knowledge of these objects is justified by the empirical science at the core of our best theory.

§1.2: A Best Theory

The first step of Quine’s argument is to settle on a single empirical theory to which we can look for our commitments. I examine two questions regarding QI.1. First, why should we find our

[1] Among them, Quines (1939a), (1939b), (1948), (1951), (1953a), (1954), (1955), (1958), (1960a), (1964), (1969b), (1976b), (1978a), (1981b), (1986b), and (1992).

commitments exclusively in an empirical theory? Second, why should we look to a single theory, which, for Quine, is a reductive physical theory?

Quine's belief that we should defer all questions about what exists to empirical science is really an expression of his naturalism, which he describes as, "[A]bandonment of the goal of a first philosophy. It sees natural science as an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal, and not in need of any justification beyond observation and the hypothetico-deductive method." (Quine (1981c) p 72)

Quine contrasts his naturalism, or relative empiricism, to the phenomenalist's radical empiricism which requires that all knowledge be reducible to claims about sense data.[2] Instead of starting with sense data and reconstructing a world of trees and persons, Quine assumes that ordinary objects exist. Further, Quine starts with an understanding of empirical science as our best account of the sense experience which gives us these ordinary objects. The job of the naturalist epistemologist, at least in part, is to describe the path from stimulus to science, rather than justify knowledge of either ordinary objects or scientific theory.[3]

Quine allows for mathematical objects by rejecting the phenomenalist's requirement for individual reductions of scientific claims to sense data. He only requires that the justifications of our scientific theory, taken as a whole, be empirical. In part, he rejects the phenomenalist's project on its own demerits, the impossibility of actually tracing the course from what appears to us in raw experience to the general and abstract claims of empirical science. More importantly, Quine rejects phenomenalism on the basis of his insight that we can adjust any theory to accommodate any evidence. The phenomenalist

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[2] In early work, Quine was agnostic between physicalism and phenomenalism. See, for example, the end of Quine (1948).

[3] We may find that our best theory rejects the existence of ordinary objects in favor of an ontology of just space-time, fields and their values at different points. In this case, we can accept only space-time points into our ontology, accounting for trees and persons on that basis. See Quine (1976b) for just such a position.

describes a system of piecemeal theory construction, where our individual experiences are each independently assessed. In contrast, Quine defends confirmation holism, that there are no justifications for particular claims independent of the justification of our entire best theory. Confirmation holism arises from an uncontroversial logical claim, that any sentence s can be assimilated without contradiction to any theory T, as long as we readjust truth values of any sentences of T that conflict with s. These adjustments may entail further adjustments, and the new theory may in the end look quite different. But we can, as a matter of logic, hold on to any sentence come what may.[4]

Since Quine relegates all questions about what exists to holistic empirical science, he rules out independent justifications for formal sciences like mathematics while allowing that mathematical knowledge can be justified as a part of our best theory.

Quine's naturalism may best be seen as a working hypothesis in the spirit of Ockham's razor. We look to our most reliable endeavor, empirical science, to tell us what there is. We bring to science a preference that it account for our entrenched esteem for ordinary experience. And we posit no more than is necessary for our best scientific theory.

Quine's naturalism, though, does not settle any questions regarding the nature or structure of empirical theory. In particular, it does not determine whether empirical science is really just physics, or whether it has various, autonomous branches. Call scientific pluralism the position which accepts various branches of science (biology, neuroscience, semantics, economics, etc.) as independent and non-reducible. The pluralist sees our best theory as some sort of amalgam of various areas of science.

In places, Quine seems to adopt scientific pluralism. He repeatedly presents the example, from Frege, of a set-theoretic definition of ancestor, as one of various,

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[4] Confirmation holism is distinct from semantic holism, the claim that meaning is a property of entire theories rather than individual terms or sentences. Semantic holism is one of the few Quinean doctrines generally irrelevant to his indispensability argument, which does not rely on a particular interpretation of the meanings of terms or sentences. For the remainder of the dissertation, 'holism' refers to confirmation holism.

> [O]ccasions which call quite directly for discourse about classes. One such occasion arises when we define ancestor in terms of parent, by Frege’s method: x is ancestor of y if x belongs to *every class* which contains y and all parents of its own members. There is this serious motive for quantification over classes; and, to an equal degree, there is a place for singular terms which name classes - such singular terms as ‘dogkind’ and ‘the class of Napoleon’s ancestors’. (Quine (1953a) p 115; see also Quine (1960a), §48 and §55; and Quine (1981b) p 14)

Quine’s references to statistical generalities also make him appear pluralistic. He countenances groups of people rather than the collections of elementary particles which would concern the reductive physicalist. “Classes [belong in our ontology] too, for whenever we count things we measure a class. If a statistical generality about populations quantifies over numbers of people, it has to quantify also over the classes whose numbers those are.” (Quine (1981b) p 14)

Even in the discussions of space and time, which do seem relevant to a physicalist theory, Quine’s examples reflect mundane applications of mathematics, rather than ones that might be used in a complete physics. “When we say, e.g. that four villages are so related to one another as to form the vertices of a square, we are talking of the arithmetical relation of the distance measurements of these villages.” (Quine (1974) p 133)

But Quine seems really to intend such talk of common uses of mathematics as merely precedental of the kind of uses that one would have in a mature physical theory. Quine is most accurately seen as a physicalist about our best scientific theory, one who believes that this theory will consist of the axioms of a completed physics. This is the position presupposed by Field’s response to the indispensability argument, Field (1980). Also, regarding Goodman’s pluralism of world versions, Quine writes, “I take Goodman’s defense of it to be that there is no reasonable intermediate point at which to end it. I would end it after the first step: physical theory.” (Quine (1978b) p 98)

Putnam also ascribes physicalism to Quine. “Quine proposes to reduce logic, mathematics, and philosophy itself to physics. The price he pays for his futurism is an enormous implausibility: virtually no one has followed Quine into the belief that the axioms of number theory, for example, are justified by their (indirect) utility in physics and natural science." (Putnam (1981d) p 183)

The question of whether to be a physicalist or a pluralist is most relevant to QI.4. The physicalist needs to find out whether uses of mathematics in physical science are eliminable. The pluralist must wonder whether one could eliminate mathematics from a broad range of scientific theories.

I proceed by taking Quine as a physicalist. But neither Quine's physicalism nor his naturalism determine the way in which we discover the commitments of our theories, which is the concern of further premises.

§1.3: Believing Our Best Theory

The second premise of Quine's argument states that our belief in a theory extends to the objects which that theory posits. I sketch Quine's argument for this premise, and examine his response to the criticism that any theory we currently believe is likely to be substantially false.

Quine's argument that there is no wedge between our belief in a theory and our beliefs in its objects is that any such distinction is double-talk. One can not arbitrarily commit only to certain elements of a theory which one accepts. If we believe a theory which says that there are ghosts, for example, then we are committed to ghosts. If we believe a theory which says that there are subvisible particles, then we are committed to subvisible particles. If our best theory posits centers of mass, or mathematical objects, then we must believe that these, too, exist.

Quine's response to Carnap's internal/external distinction relies on the double-talk criticism. The claim that there is a prime number between four and six seems to entail that a number exists. Carnap proposed that we can accept that five is prime, since that is an internal result within mathematics, without making the further step of accepting that numbers exist, which is properly speaking an external question about whether to adopt number language. Quine responds that if we accept that five is prime, then we are committed to its existence. If we reject number language, we can no longer claim that five is prime, since there are no numbers to be prime.

Putnam makes the double-talk criticism explicitly. "It is silly to agree that a reason for believing that p warrants accepting p in all scientific circumstances, and then to add 'but even so it is not *good enough*'." (Putnam, p 356)

Field also makes the double-talk criticism, specifically regarding mathematics. "If one *just* advocates fictionalism about a portion of mathematics, without showing how that part of mathematics is dispensable in applications, then one is engaging in intellectual doublethink..." (Field (1980) p 2)

QI.1 and QI.2 together entail that we should believe in the objects that our currently best theory says exist. These posits are made together, in the same way. Any evidence applies to the whole theory, which produces its yield uniformly. Quine thus makes no distinction between justifications of observable and unobservable objects, or between mathematical and concrete objects. All objects, trees and electrons and sets, are equally posits of our best theory, to be taken equally seriously. Call this aspect of Quinean metaphysics, that all posits are made together with equal seriousness, homogeny. Homogeny is a manifestation of Quine's insistence that there is only a single way of knowing of anything. We receive sensory stimulus and construct a single theory to account for it. What exists, all objects, are the posits of that theory. "To call a posit a posit is not to patronize it." (Quine (1960a) p 22)

Homogeny permits Quine to avoid some criticism, as Charles Parsons notes. "In the case of abstract entities, certain protests against Platonism become irrelevant. There is no mysterious 'realm' of, say, sets in the sense that they need to have anything akin to location, and our knowledge of them is not based on any mysterious kind of 'seeing' into such a realm. This 'demythologizing' of the existence of abstract entities is one of Quine's important contributions to philosophy..." (Parsons (1983) pp 377-8)

But, there will be conflict between our currently best theory and ideal theories future science will produce. Ideal theories are, of course, not now available. What exists does not vary with our best theory. Thus, any current expression of our commitments is at best speculative. How can we hope to determine what exists on the basis of a continually changing science?

Facing the problem of the progress of science, one might mis-interpret the indispensability argument too strongly:

(SQI) SQI.1: What exists is what our ideal theory will quantify over.
SQI.2: Our ideal theory will quantify over mathematical objects.
SQI.C: Mathematical objects exist.

This stronger indispensability argument has a more compelling conclusion, but can not truly be ascribed to Quine. First, and most importantly, we do not know what our ideal theory will be, so we lack support for SQI.2. Additionally, Quine has several reasons for denying that we have any categorical knowledge of what exists, as SQI.C claims we do. Some of these reasons emerge in his later work on ontological relativity. Empirical theories are generally underdetermined by evidence, so we have to choose a best theory from among empirically equivalent options. Also, the objects to which a theory commits are found by examining models of our theories and our best theory will have multiple conflicting models. Thus, we can only determine a theory’s commitments relative to a given model. The models themselves are subject to interpretation, as well.

We must have some skepticism toward our currently best theory, if only due to an inductive awareness of the transience of such theories. Applying this skepticism, one who denies Quine’s indispensability argument might say that our best theory commits to mathematical objects, but we are not really committed to our best theory. Such skepticism, though, must be strictly speculative, and unavailable to Quine. For Quine, we are adrift on Neurath’s boat, with no external, meta-scientific perspective from which to judge our best theory.

QI.1 and QI.2 say that we should believe that the posits of our best theory exist. They do not tell us how to determine what those posits are, which is the job of the next premise.

§1.4: Quine's Procedure for Determining Ontic Commitments

The third step of Quine's argument is an appeal to his general procedure for determining the ontic commitments of any theory. Any one who wishes to know what to believe exists, and in particular whether to believe that mathematical objects exist, needs a general method for determining ontic commitment. There are many possible criteria. Most casually, we might rely on our brute observations. But our senses are limited, and the content of experience is ambiguous.

Another method would involve looking at our ordinary language. Perhaps the referents of our common singular terms are what exist. But ordinary language is also misleading and incomplete.

Quine provides a simple and broadly applicable procedure for determining the ontic commitments of any theory.

(QP) QP.1: Choose a theory.
QP.2: Regiment that theory in first-order logic with identity.
QP.3: Examine the domain of quantification of the theory to see what objects the theory needs to come out as true.

I have discussed the application of QP.1 to the indispensability argument in the previous two sections. But, QP applies to any theory. Theories which refer to ghosts, caloric, and God are equally amenable of Quine's general procedure. In the next two subsections, I discuss Quine's arguments for QP.2 and QP.3.

§1.4.1: First-Order Logic

In this section, I sketch Quine's defense of first-order logic as his canonical language. Quine credits first-order logic with extensionality, efficiency, and elegance, convenience, simplicity, and beauty. More concretely, Quine stresses first-order logic's unification of the referential apparatus of ordinary, and scientific, language. The existential quantifier is a natural cognate of 'there is', and Quine proposes that all existence claims can and should be made by existential sentences of first-order logic, which also has various technical virtues which make it an attractive language. He also writes of his canonical language, "*The* reason for taking the regimented notation as touchstone is that it is explicit referentially, whereas other notations, having other aims, may be vague on the point." (Quine (1986c) p 534, emphasis added)

First-order logic with identity is a formal language of predicates and variables, logical connectives, some optional punctuation, quantifiers, and an identity predicate.[5] Quine argues that we can use this language as canonical since we can use it to express anything we need to say. "The doctrine is that all traits of reality worthy of the name can be set down in an idiom of this austere form if in any idiom." (Quine (1960a) p 228)

We should take first-order logic as canonical only if: A) We need a single canonical language; B) It really is adequate; and C) There is no other adequate language. In Chapter 2, I will deny each of these clauses. Here, I sketch Quine's reasons for holding them.

Condition A arises almost without argument from QI.1 and QI.2. One of Quine's most striking and important innovations was his linking of our concerns when constructing formal theory, when regimenting, with general existence questions. When we regiment our correct scientific theory correctly, we will know what exists. "The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality." (Quine (1960a) p 161)

Quine's arguments for condition B consist in showing how first-order logic is useful as a tool for semantic ascent. It provides a framework for settling disagreements over ontic commitments, for with it we can deny the existence of objects without seeming to commit to them. Thus, $\sim\exists xPx$, where 'P' is a predicate standing for the property of being Pegasus, carries with it no implication that Pegasus exists. On the contrary, some one who holds that the meaning of a name is its referent must confront the puzzle of how 'Pegasus does not exist' can have meaning. Still, the burden of showing that first-order logic is adequate is greater than showing its utility in some contexts, a topic to which I return in Chapter 2. I

[5] Henceforth, I will call Quine's canonical language merely 'first-order logic'.

proceed to condition C, Quine's argument that no other language is adequate for canonical purposes. I focus on Quine's arguments against languages with names, and against higher-order languages.

Quine contrasts the use of the first-order quantifiers to express reference with the use of languages with names. Names may be non-referential, like 'Pegasus'. There are not enough names for distant stars and real numbers. Reference may also be found in pronouns, which diffuses the matter. Unifying reference in the first-order quantifiers, rather than using names, simplifies the task. Instead of looking for real names among the various general and singular terms, pronouns, and proper nouns, we can look only to the quantifiers of the theory.

We are forced to choose between names and quantifiers, since we can not just include names in a language with quantifiers. Consider the following derivation, in a language which includes both, which entails that anything named exists:

1. $\sim(\exists x)x=a$	Assumption, for indirect proof
2. $(\forall x)x=x$	Principle of identity
3. $(\forall x)\sim x=a$	1, Change of quantifier rule
4. $a=a$	2, UI
5. $\sim a=a$	3, UI
6. $(\exists x)\ x=a$	1-5, Indirect proof [6]

The derivation should be taken as a reductio on the adequacy of any language which includes both names and quantifiers. If we accept Quine's reasons for preferring a canonical language which includes quantifiers, then we are left to choose among first-order and higher-order logics.[7] First-order logic has a variety of characteristics which higher-order logics lack. In first-order logic, a variety of definitions of logical truth concur: in terms of logical structure, substitution of sentences or of terms,

[6] I owe the derivation to David Rosenthal.

[7] I do not accept Quine's reasons for preferring quantifiers over names. See Chapter 2, Part 1.

satisfaction by models, and proof.[8] First-order logic is complete, in the sense that any valid formula is provable. Every consistent first-order theory has a model. First-order logic is compact, which means that any set of first-order axioms will be consistent if every finite subset of that set is consistent. It admits of both upward and downward Löwenheim-Skolem features, which mean that every theory which has an infinite model will have a model of every infinite cardinality (upward) and that every theory which has an infinite model of any cardinality will have a denumerable model (downward). All of these properties fail in second-order logic.[9] Furthermore, second-order logic quantifies over properties. If we take its quantifiers as indicating existence, then it yields too many objects even for the needs of mathematics. As with Quine's arguments for preferring quantifiers to languages with names, I return critically to this matter in Chapter 2.

§1.4.2: The Domain of Quantification

QP.1 and QP.2 give us a first-order theory. Now, we must determine its commitments. Reading existential claims seems prima facie quite straightforward. Consider the null set axiom, $(\exists x)(\forall y)\sim(y \varepsilon x)$, which, taken at face value, states that the null set exists. We can not conclude that objects exist directly from existential claims. Instead, we must figure out what objects the sentences of the theory require for

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[8] See Quine (1986a) p 79, and p 87. The importance of logical truth is emphasized by Quine's hesitance to include identity as a logical particle. The class of logical truths shifts when we substitute other predicates for the identity predicate. Quine includes identity, despite the loss of logical truth via substitution, since identity applies generally, and there are complete proof procedures for first-order logic with identity; see Quine (1960a) §24. The identity predicate facilitates working with names, e.g. to characterize a uniqueness clause essential to Russell's treatment of empty names. We can also perform elementary arithmetic tasks without appeal to mathematical objects. Lastly, Quine takes identity as a basis for ontology: "No entity without identity." (Quine (1958) p 23) Strictly speaking, Quine's use of identity for dividing reference need not entail its inclusion as an element of the canonical language. In fact, Quine urges us to think of identity as definable in terms of the extensions of predicates. This assumes, of course, a finite stock of predicates. See Quine (1986a) p 64.

[9] See Mendelson (1997) p 377.

their truth. We ascend to a metalanguage to construct a domain of quantification in which we find values for all variables of the object theory. "To be is to be a value of a variable." (Quine (1939a) p 50, among others)

The move to a metalanguage means that we do not directly interpret first-order theories to find ontic commitments. We look to their models. Quine's reasons for examining models, rather than the theorems directly, is simply formal. We find our commitments in examining existential quantifications, but quantifications bind variables which are not themselves the things we think exist. Nor are their substituends what exist; these may be taken as names of the things that exist. Variables take as values the things that exist, and these values are located in the domain of the theory.

One reason to favor Quine's procedure is because it can prevent prejudging what exists. Call this the neutrality of Quine's method. On his view, we construct scientific theory without prior determination of what exists. Scientists take the evidence and the theory wherever it leads them. They balance formal considerations, like the elegance of the mathematics involved, with an attempt to account for the broadest empirical evidence. The more comprehensive and elegant the theory, the more we are compelled to believe it, even if it tells us that the world is not the way we thought it is. If the theory yields a heliocentric model of the solar system, or the bending of rays of light, then we are committed to heliocentrism or bent light rays. Our ontic commitments are the byproducts of this neutral process.

The method I am ascribing to Quine, especially its neutrality, may seem a bit like a caricature. For, it makes the determination of our commitments the result of blind construction of scientific theory. This criticism is fundamentally correct, but it is a criticism of Quine's method itself, and not of my interpretation.

The indispensability argument, which Quine clearly holds, depends on the method I have ascribed to Quine. Without QP, we are free to interpret quantifications over mathematical objects instrumentally, as not indicating commitments to them. This fact supports my interpretation. Further, Quine's naturalism and his presumption of parsimony makes him skeptical of abstract objects.[10] Unless we ascribe to Quine the neutral method I have presented, there is no reason for him to yield his skepticism. If Quine abandoned neutrality, he could easily see the indispensability argument as a reductio on his procedure for determining commitment. His strong pre-theoretic commitment to nominalism's austere landscapes should take over. The neutrality of Quine's method is what ensures that he does not stack the deck against mathematical objects.

Field's response to the indispensability argument, as I show in the next part of this chapter, is a response to the method I described. So, even if the argument I have presented is not in fact Quine's, it is an important one, and worth consideration.

§1.5: Mathematization

The final step of Quine's indispensability argument involves simply looking at the domain of the theory we have constructed and regimented. We discover that the theory includes, in the casting of physical laws, functions, sets, and numbers. For example, consider Coulomb's Law: $F = k\,|q_1 q_2| / r^2$. This law states that the electromagnetic force between two charged particles is proportional to the charges on the particles and, inversely, to the distance between them.

Regimenting Coulomb's Law, or any sentence of physics, all the way down into first-order logic would make it quite complicated. Here is a first step, using 'Px' for 'x is a charged particle'.

(CL) $\forall x \forall y \{(Px \wedge Py) \supset (\exists f)[<q(x), q(y), d(x,y), k, F> \mid F = (k \cdot |q(x) \cdot q(y)|) / d(x,y)^2\}$

This regimentation is incomplete, but it suffices to give the idea of the commitments of the law. Besides the charged particles over which the universal quantifiers in front range, there is an existential quantification over a function, f. Furthermore, this function maps numbers (the Coulomb's Law constant,

[10] See Quine and Goodman (1947) for his attempt to avoid mathematical objects.

and measurements of charge and distance) to other numbers (measurements of force between the particles).

In order to ensure that there are enough sets to construct these numbers and functions, and in order to round out the theory, which may be justified by considerations of simplicity, our ideal theory will include set-theoretic axioms, perhaps those of Zermelo-Fraenkel set theory, ZF. Quine, unsatisfied with ZF for its piecemeal, type-theoretic avoidance of paradox, formulated alternative systems NF and ML, which extended NF by adding classes. We can derive from the axioms of any of these set theories a vast universe of sets. So, CL contains or entails several existential claims.

CL, with its mathematical commitments, is representative of the kind of physical law that motivates Quine's indispensability argument. In Chapter 2, I will present considerations against the version of the argument I have presented here. First, in the last section of Part 1 of this chapter, I show that attempts to avoid commitments to mathematical objects by reinterpreting the mathematical claims of science modally do not succeed as responses to QI. Then, in Part 2 of this chapter, I consider the most well-known and fecund response to the argument presented here, Field's project.

§1.6: Modal Reinterpretations

Typically, criticisms of indispensability arguments deny QI.4 and attempt to show that science can be recast to avoid commitments to mathematical objects. These are dispensabilist projects. In this section, I show how QI withstands modal dispensabilist projects.

The dispensabilist is likely to be motivated by mathematical nominalism, which takes diverse forms, many of them interesting independently of the indispensability argument.[11] The exhaustive Burgess and Rosen (1997) elegantly unifies many significant dispensabilist strategies within a flexible

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[11] I use 'nominalism' for the denial that mathematical objects exist, and 'dispensabilism' for granting QI.1 - QI.3 while denying QI.4.

technical framework. Most of these strategies include significant appeals to modality, or other extensions of first-order logic, rejecting Quine’s canonical language. If a reformulation of scientific theory is to eliminate commitments to abstract objects, our real commitments must be found in the reformulated theory. But theories which appeal to modality or extended logic can not rely on Quine’s arguments that we find our commitments by looking at our regimented theory. Such arguments only apply to Quine’s canonical language, not to any kind of regimentation. Thus, reformulations which reject the indispensabilist’s method for determining commitment do not succeed as dispensabilist strategies.

In particular, almost all of the reformulations considered by Burgess and Rosen swap mathematical objects for possibilities. The complexity and sophistication of these attempts does not mask an underlying modal profligacy. The addition of modal logic mitigates the significance of the elimination of quantification over mathematical objects. “Avoidance of modalities is as strong a reason for an abstract ontology as I can well imagine.” (Quine (1986b) p 397)

The problem of reducing the ontic commitments of a theory by extending one’s logic may be demonstrated quite clearly. One method involves predicate functors.[12] One can construct a first-order language including predicate functors and predicates, but no variables. The first-order quantificational sentence ‘$\exists xPx$’, for ‘There are prime numbers’, has a predicate functor correlate, say, ‘ζP’. Given a first-order theory, one can introduce functors to replace logical connectives, and then to replace quantifiers and variables. Burgess and Rosen present the following transformation of, ‘Whatever lives, changes’.

$\sim\exists x(Fx \bullet \sim Gx)$	Translation into first-order logic
$\sim\exists x(Fx \bullet (\nu G)x)$	Introducing a functor for negation
$\sim\exists x(\kappa F(\nu G))xx$	Introducing a functor for conjunction
$\sim\exists x(\rho(\kappa F(\nu G)))x$	Eliminating one variable using a functor for ‘the same thing’
$\sim\zeta(\rho(\kappa F(\nu G))))$	Eliminating the quantifier and bound variable for a functor
$\nu(\zeta(\rho(\kappa F(\nu G))))$	Negation, again.

[12] Quine first explored predicate functors, in Quine (1960c), as a way of explicating quantification in sententialist terms, replacing variables (pronouns) with sentential operators. See also Quine (1960b), Quine (1970), and Quine (1982), §45; and Bacon (1985) for deduction rules and completeness results. A summary is provided in Burgess and Rosen (1997) pp 186-7.

Predicate functors allow us to eliminate quantification completely. A scientific theory reformulated in the language of predicate functors has no quantifications over mathematical objects. "If it is permitted to help oneself to whatever logical apparatus one wants, while ignoring the usual definitions and giving no other explanations, then not only a nominalistic but a monistic and even a nihilistic reformulation can very easily be given..." (Burgess and Rosen (1997) p 186)

No one should consider a translation of standard science into the language of predicate functors a demonstration that mathematical objects do not exist. Adopting predicate functors, one changes the way in which commitments are to be found within a theory. The example shows that we must concern ourselves with the logic used in dispensabilist projects.

The question at hand is whether modal reformulations of science can succeed as dispensabilist projects.[13] Modality is typically construed in terms of possible worlds. Philosophers have become increasingly comfortable with possible worlds in the wake of developments in model theory for modal logic, especially Kripke models. In a Kripke model, '$\Diamond\mathscr{F}$' is true in a world just in case there is an accessible possible world in which $\mathscr{F}$ is true.

Different modal logics reflect varying degrees of modal commitment. Putnam's modalism is among the most profligate. For Putnam, a statement is possible just in case it is consistent with any true mathematical theory. He thus does not eliminate mathematics. "Something very akin to mathematical truth (and therefore to mathematical existence) is being sneaked into Putnam's possibility operator." (Field (1984) p 85)

Burgess and Rosen's pure modal strategy, based on Charles Chihara's constructabilism,[14] relies on a novel version of modal logic. Possible inscriptions capable of coding real numbers replace

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[13] More precisely, we are considering modal reinterpretations of the mathematical axioms used in scientific theories.

[14] See Chihara (1990).

mathematical objects. This reformulation requires what Burgess and Rosen call metaphysical modality as opposed to metalogical modality. Metaphysical modal logics are rigid, which means that iterated modal operators are redundant; rigid modal logics admit no double subjunctive moods. Most modal logics of interest are metalogical, in which iterations of modal operators create different formulas with different implications. Metalogical modality makes weak modal commitments since it interprets possibility as non-contradiction, and may be palatable to those wary of modal logics.[15] In contrast, metaphysical modality commits to what might have been the case if the world had been other than it is: what kinds of objects may exist together, and how a thing would be if other things had existed. The pure modalist must claim that some inscription, ‘a’, if it were to exist, would have marked more than another inscription ‘b’, were it to exist.

Metaphysical modalities are just as epistemically intractable as abstract objects. In fact, they may be taken to be abstract since they are isolated from us outside of space and time. The use of modal logic undermines the claim that we should look to science to determine our commitments. In fact, any appeal to possible worlds is problematic for the dispensabilist. Modal claims conflict with the indispensabilist’s parsimony.

If a mature science required substantial modality for purposes other than the elimination of mathematical objects (to formulate laws which apply to counterfactuals, say) then modal reformulations of science might be worth adopting. This is an open possibility, despite Quine’s skepticism, but not one that I am prepared to examine in this dissertation. I will thus ignore modal reformulations and other dispensabilist strategies which rely on significantly extending logic.[16]

As we deviate from the canonical language, the claim that we find our commitments in the

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[15] Field uses the weaker modality with his modal operator.

[16] Specifically, I ignore those of Chihara, both the early predicativism and the later constructibilism, Bostock’s Russellian project, Hodes’s Fregean project, and those of Bigelow, Putnam, and Hellman, among others. See Burgess and Rosen (1997), §IIIB.

quantifications of a theory fades.

> [I]t can be seen that there is something dubious about the practice of just helping oneself to whatever logical apparatus one pleases for purposes of nominalistic reconstruction while ignoring any customary definitions that would make the apparatus nominalistically unpalatable: for by doing so, one can make the task of nominalistic reconstruction absolutely trivial – and so absolutely uninteresting. (Burgess and Rosen (1997) p 175)

Quine chose first-order logic as his canonical language for its neutrality. Dispensabilist reformulations which rely on extensions of logic which violate that neutrality can not be seen as legitimate responses to the indispensability argument. QI thus withstands this kind of response. In the next part of this chapter, I show how Field's *Science Without Numbers* project, and related ones, present a more significant challenge to Quine's argument.

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